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TOTAL COLORING REGULAR DOMINATION in FUZZY GRAPH

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Abstract

In this paper we discuss the concept of total coloring Regular domination in fuzzy graph. We determine the chromatic number χ^{rf} for a Regular domination fuzzy graph G_k, with fuzzy set of vertices and fuzzy set of edges in terms of family of fuzzy sets.

Keywords:

Regular fuzzy graph,
Domination in fuzzy graph,
Regular domination in fuzzy graph,
Fuzzy total coloring,
Chromatic Number.

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I.INTRODUCTION

As a advancement fuzzy coloring of a fuzzy graph was defined by authors Eslahchi and Onagh in 2004, and later developed by them as fuzzy vertex coloring [1] in 2006. This fuzzy vertex coloring was extended to fuzzy total coloring in terms of family of fuzzy sets by Lavanya. S and Sattanathan.R [2]. On Regular Fuzzy Graphs was defined by the authors A. Nagoor Gani and K. Radha[3].

In this paper we discuss on k −fuzzy total coloring of a Regular domination fuzzy graph by taking fuzzy sets of vertices and edges. In section 2 we review the basic definitions of fuzzy sets and other basic definitions of fuzzy graphs. In section 3 we introduce the definition of total coloring and chromatic number regular domination in fuzzy graph and their example and theorem.

II. PRELIMINARYS

Definition: 2.1

Let V be a finite nonempty set. The triple $G = (V, \sigma, \mu)$ is called a *fuzzy graph* on V where σ and μ are fuzzy sets on V and E, respectively, such that $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$ and $uv \in E$. For fuzzy graph $G = (V, \sigma, \mu)$ the elements V and E are called set of vertices and set of edges of G respectively.

Definition: 2.2

Two vertices u and v in G are called *adjacent*if $(1/2)[\sigma(u) \wedge \sigma(v)] \leq \mu(uv)$.

Definition: 2.3

Two edges $v_i v_j$ and $v_i v_k$ are said to be *incident* if $2\{\mu(v_i v_j) \wedge \mu(v_i v)\} \leq \sigma(v_i)$ for $i = 1,2, \dots \dots |v|$ and $1 \leq j, k \leq |v|$.

Definition: 2.4

Let G:(σ,μ) be a fuzzy graph. The *degree* of a vertex $u$is $d_G(u) = \sum_{u \neq v} \mu(uv)$. Since $\mu(uv) > 0$ for $uv \in E$ and $\mu(uv) = 0$for $uv \notin E$, this is equivalent to $d_G(u) = \sum_{uv \in E} \mu(uv)$. The minimum degree of G is $\delta(G) = \wedge\{d(v)/v \in V\}$. The maximum degree of G is $\Delta(G) = \vee\{d(v)/v \in V\}$.

Definition: 2.5

Let $G = (V, E)$ be a graph. A subset S of V is called a *dominating set* in G is every vertex in $V \backslash S$ is adjacent to some vertex in S.

Definition: 2.6

A fuzzy graph $G = (V, \sigma, \mu)$ is called a *completefuzzy graph* if $(uv) = \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$ and $uv \in E$.

Definition: 2.7

An assignment of colours to the vertices of a graph so that no two adjacent vertices get the same colour is called a *colouring* of the graph.

Definition: 2.8

The *chromatic number* $\chi(G)$ of a graph Gs is the minimum number of colours needed to colour.

Definition: 2.9

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Let $G = (\sigma, \mu)$ be a fuzzy graph on V. Let $x, y \in V$. We say that x dominates y in Gif $\mu(xy) = \sigma(x) \wedge \sigma(y)$. A subset S of V is called a *dominating set* in G if for every $v \notin S$, there exists $u \in S$ such that u dominates v.

Definition: 2.10

The minimum fuzzy cardinality of a dominating set in G is called the *domination number* of G and is denoted by
$\gamma(G)$or γ.

Definition: 2.11

Let $G: (\sigma, \mu)$ be a fuzzy graph on $G^*: (V, E)$. If $d_G(v) = k$ for all $v \in V$, (i.e) if each vertex has same degree k, then G is said to be a *regular fuzzy graph of degree* k *or a* k *−regular fuzzy graph.*

Definition: 2.12

Let $G: (\sigma, \mu)$ be a fuzzy graph on G^*. The total degree of a vertex $u \in V$ is defined by $td_G(u) = \sum_{u \neq v} \mu(uv) + \sigma(u) = \sum_{uv \in E} \mu(uv) + \sigma(u) = d_G(u) + \sigma(u)$. If each vertex of G has same total degree k, then G is said to be a *totally regular fuzzy graph of total degree* k *or* k *−totally regular fuzzy graph.*

Definition: 2.13

A family $\Gamma = \{\gamma_1, \dots \dots, \gamma_k\}$ of fuzzy sets on V is called a k *−fuzzy coloring* of $G = (V, \sigma, \mu)$ if

(a) $\vee \Gamma = \sigma$,

(b) $\gamma_i \wedge \gamma_j = 0$,

(c) for every strong edge xy of G, $min\{\gamma_i(x), \gamma_i(y)\} = 0$ $(1 \leq i \leq k)$.

Definition: 2.14

A family $\Gamma = \{\gamma_1, \gamma_2, \dots \dots \gamma_k\}$ of fuzzy sets on $V \cup E$ is called a k −fuzzy total coloring of $G = (V, \sigma, \mu)$ if

(a) $max_i\, \gamma_i(v) = \sigma(v)$ for all $v \in V$ and $max_i\, \gamma_i(uv) = \mu(uv)$ for all edge $uv \in E$,

(b) $\gamma_i \wedge \gamma_j = 0$,

(c) For every adjacent vertices u, v of $\min\{\gamma_i(u), \gamma_i(v)\} = 0$ and for every incident edges $\min\{\gamma_i(v_j v_k)/v_j v_k$ are set of incident edges from the vertex v_j, $j = 1, 2, \dots |v|\}$.

Definition: 2.15

The least value of k for which G has a k −fuzzy total coloring, denoted by $\chi_T(G)$, is called the *fuzzy total chromatic number* of G.

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III TOTAL COLORING REGULAR DOMINATION IN FUZZY GRAPH

We extend the definition of total coloring regular domination in fuzzy graph in the definition given below. Since we deal with regular domination fuzzy graph for which $d_G(v) = k$, the definition can be stated as follows.

Definition: 3.1

Let $G:(V,\sigma,\mu)$ be a fuzzy graph and S be a subset of V. Then a family $\Gamma = \{\gamma_1, \gamma_2, \ldots \gamma_k\}$ of fuzzy sets $V \cup E$ is called a *total coloring Regular domination in fuzzy graph* if,

(a) All the vertices in S has the same degree, [3]
(b) Every vertex in $V - S$ is adjacent to some vertex in S, [4]
(c) $\vee \gamma_i(v) = \sigma(v)$ for all $u, v \in V$, [2]
(d) $\gamma_i \wedge \gamma_j = 0$,
(e) For every adjacent vertices u, v of $\min\{\gamma_i(u), \gamma_i(v)\} = 0$ and for every incident edges uv on vertex $u \in V$ of G, $\wedge\{\gamma_i(uv)\} = 0$.

Definition: 3.2

The least value of k for which G has a total coloring regular domination in fuzzy graph, denoted by $\chi_T^{rf}(G)$ is called the fuzzy total chromatic number of G [6].

Example: 3.3

Consider the fig-1, a regular domination fuzzy graph $G_k = (V, \sigma, \mu)$ with vertex set $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and edge set $E = \{v_i v_j / ij = 12,16,23,34,45,56\}$ the membership functions are defined as follows:

$$\sigma(v_i) = \begin{cases} 0.2, for\ i = 1 \\ 0.7, for\ i = 2 \\ 0.5, for\ i = 3 \\ 0.4, for\ i = 4 \\ 0.6, for\ i = 5 \\ 0.3, for\ i = 6 \end{cases}$$

$$\mu(v_i v_j) = \begin{cases} 0.2, for\ ij = 12. \\ 0.5, for\ ij = 23. \\ 0.1, for\ ij = 16,34. \\ 0.4, for\ ij = 45. \\ 0.3, for\ ij = 56. \end{cases}$$

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We see that the membership functions satisfy the definition of regular domination fuzzy graph. In fig-1 $S = \{v_2, v_5\}$ is a dominating set.

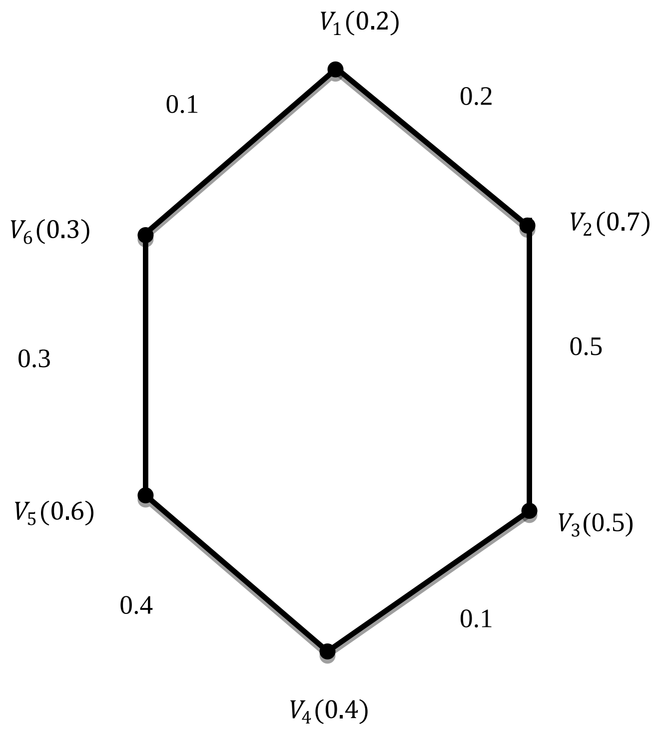

Figure-1

Let $\Gamma = \{\gamma_1, \gamma_2\}$ be a family of fuzzy sets defined on $V \cup E$ as follows:

$$\gamma_1(V_i) = \begin{cases} 0.2, & i = 1 \\ 0.5, & i = 3 \\ 0.6, & i = 5 \\ 0, & otherwise \end{cases} \qquad \gamma_1(v_i v_j) = \begin{cases} 0.2, & ij = 12 \\ 0.1, & ij = 34 \\ 0.3, & ij = 56 \\ 0, & otherwise \end{cases}$$

$$\gamma_2(v_i) = \begin{cases} 0.7, & i = 2 \\ 0.4, & i = 4 \\ 0.3, & i = 6 \\ 0, & otherwise \end{cases} \qquad \gamma_2(v_i v_j) = \begin{cases} 0.5, & ij = 23 \\ 0.4, & ij = 45 \\ 0.1, & ij = 16 \\ 0, & otherwise \end{cases}$$

Hence the family $\Gamma=\{\gamma_1,\gamma_2\}$ satisfies our definition of total coloring regular domination in fuzzy graph. From the table given below, we can see the values γ_1,γ_2 clearly. Hence in this case the total chromatic number $\chi_T^{rf}(G)$is 2.

Table-1

Vertices and Edges	γ_1	γ_2	max
1	0.2	0	0.2
2	0	0.7	0.7
3	0.5	0	0.5
4	0	0.4	0.4
5	0.6	0	0.6
6	0	0.3	0.3
12	0.2	0	0.2
16	0	0.1	0.1
23	0	0.5	0.5
34	0.1	0	0.1
45	0	0.4	0.4
56	0.3	0	0.3

Threoem: 3.4

For a Regular Domination in fuzzy graph $G:(V,\sigma,\mu)$then $\chi(G)=\chi^{rf}(G)$.

Proof:

Let $G=(V,\sigma,\mu)$ be a regular domination in fuzzy graph on $n$vertices, $\{u_1,u_2,\ldots.u_n\}$.

Let $\chi^{rf}(G)=k$.

$\Leftrightarrow \Gamma=\{\gamma_1,\ldots\ldots,\gamma_k\}$is a $k-$fuzzy coloring and let C_j be the color assigned to vertices in $\gamma_j{}^*, j=1,2,\ldots.k$.

$\Leftrightarrow\{\gamma_1,\ldots\ldots,\gamma_k\}$is a family of fuzzy sets where,

$\gamma_j(u_i)=\{(u_j,\sigma(u_j))\cup(u_i,\sigma(u_i))/\mu(u_i,u_j)=0, i\neq j\}$which follows from (iii) and (v) of definition (3.1).

Also $\bigcup_{j=1}^{k}\gamma_j{}^*=V$ and $\gamma_i{}^*\cap\gamma_j{}^*=\phi$, $i\neq j$ which follows from (iv) of definition (3.1).

$\Leftrightarrow \gamma_j{}^*$ is an independent set of vertices and edges,((i.e) no two vertices in $\gamma_j{}^*$ are adjacent and no two edges in $\gamma_j{}^*$ are incident) for each $j=1,2,\ldots\ldots k$.

$\Leftrightarrow \chi(G^*) = k$, G^* is the underlying crisp graph of G.

Now,

$\Leftrightarrow \chi(G^*) = \chi(G_t) = k$ where $t = \min\{\alpha / \alpha \in L\}$.

Since, $\chi(G) = \max\{(\chi_\alpha / \alpha \in L$ where $\chi_\alpha = \chi(G_\alpha)$.

Therefore, $\chi(G_\alpha) = k$,

$\max\left\{\chi_\alpha / \alpha \in L\right\} = k$.

$\Leftrightarrow \chi(G) = k$.

Hence the proof.

V. CONCLUSIONS

The concept of $k-$fuzzy total coloring and chromatic number of a fuzzy graph are analized. In this paper the total coloring and chromatic number regular domination in fuzzy graph are introduced and determined the example and theorem.

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